Global MHD Simulation Using Modified M-I Coupling Algorithm

H. Nakata
Graduate School of Science and Technology, Chiba University, 1-33 Yayoi-ryo, Inage-ku, Chiba 263-8522, Japan
nakata@faculty.chiba-u.jp

A. Yoshikawa and T. Tanaka
Department of Geophysics, Kyushu University, 6-10-1 Hakozaki, Higashi-ku, Fukuoka 812-8581, Japan

1. Introduction
Recent global MHD simulations have adopted the following boundary condition of the inner magnetosphere. First, field-aligned currents (FACs) are determined by magnetic perturbations at the inner boundary of the magnetosphere. Mapping the FACs into the ionosphere along the main magnetic field, the ionospheric electric potentials are calculated using the equation of current continuity,

$$ -\nabla (\Sigma \nabla \phi) = j_\parallel \sin I, \quad \text{(1)} $$

where $j_\parallel$ is FAC, $I$ is the inclination of the main magnetic field, $\Sigma$ is ionospheric conductivity tensor, and $\phi$ is the ionospheric electric potential. The resultant ionospheric electric potentials are mapped out to the inner boundary of the magnetosphere. However, the FACs and the potentials determined using this M-I coupling algorithm are not always satisfied to this boundary condition at the ionosphere and the inner boundary, simultaneously. In this study, we propose a modified self-consistent process of magnetosphere-ionosphere coupling and show the results of the global MHD simulation using this algorithm.

2. Modified M-I coupling algorithm
In the modified process of the magnetosphere-ionosphere coupling, we separate the correct values of the FAC and the potential at the inner boundary into three components as follows:

$$ j_{\parallel|M} = j_{\parallel|M}^0 + \delta j_{\parallel|M-I} + \delta j_{\parallel|M-I}, \quad \text{(2)} $$

$$ \phi_M = \phi_M^0 + \delta \phi_{M-I} + \delta \phi_{M-I}, \quad \text{(3)} $$

where $(j_{\parallel|M}, \phi_M)$ are the correct values of the FAC and the electric potential in the inner boundary. $(j_{\parallel|M}^0, \phi_M^0)$ are the correct values at the previous step. $(\delta j_{\parallel|M-I}, \delta \phi_{M-I})$ are perturbed components propagating from the magnetosphere, and $(\delta j_{\parallel|M-I}, \delta \phi_{M-I})$ are the additional components produced by M-I coupling. Thus, all MHD schemes calculate $(\delta j_{\parallel|M-I}, \delta \phi_{M-I})$ from $(j_{\parallel|M}^0, \phi_M^0)$ whether the effect of M-I coupling is included or not. In this study, these components which the MHD schemes calculate are described as

$$ j_{\parallel|MHD} = j_{\parallel|M}^0 + \delta j_{\parallel|M-I}, \quad \text{(4)} $$

$$ \phi_{MHD} = \phi_M^0 + \delta \phi_{M-I}. \quad \text{(5)} $$

Since the correct values, $(j_{\parallel|M}, \phi_M), (j_{\parallel|M}^0, \phi_M^0)$, satisfy equation (1), $(\delta j_{\parallel|M-I}, \delta \phi_{M-I})$ and $(\delta j_{\parallel|M-I}, \delta \phi_{M-I})$ are balanced. Therefore, we can treat that M-I coupling is regarded as the problem of wave reflection. Assuming that these components are associated with the shear Alfvén waves, we have the relation equations as follows;

$$ \delta j_{\parallel|M-I} = V_A \nabla (\varepsilon_A \nabla \delta \phi_{M-I}), \quad \text{(6)} $$

$$ \delta \phi_{M-I} = -V_A \nabla (\varepsilon_A \nabla \delta \phi_{M-I}). \quad \text{(7)} $$

Here, $V_A$ is the Alfvén speed and $\varepsilon_A$ is the permittivity in the MHD region ($\varepsilon_A = 1/\mu_0 V_A^2$). Finally, we have

$$ -\nabla (\Sigma \nabla (\phi_{MHD} + \delta \phi_{M-I})) = (j_{\parallel|MHD} - V_A \nabla (\varepsilon_A \nabla \delta \phi_{M-I})) \sin I. \quad \text{(8)} $$

In this equation, the undetermined parameter is only $\delta \phi_{M-I}$, which is produced by the effect of M-I coupling.

In the present study, Equation (8) is adopted as M-I coupling equation instead of Equation (1). The procedure of the modified M-I coupling algorithm is as follows. Giving $(j_{\parallel|MHD}, \phi_{MHD})$ by the MHD scheme, $\delta \phi_{M-I}$ is determined using Equation (8). $\delta j_{\parallel|M-I}$ is also determined from $\delta \phi_{M-I}$ using equation (7). Mapping $(j_{\parallel|M}, \phi_{M})$ onto the inner boundary, $(j_{\parallel|MHD}, \phi_{MHD})$ is replaced by $(j_{\parallel|MHD} + \delta j_{\parallel|M-I}, \phi_{MHD} + \delta \phi_{M-I})$. Then the MHD parameters at the next step are calculated by the MHD scheme.
3. Results

Here, we describe the ionospheric electric potentials determined by the global MHD simulation scheme with the present algorithm. To examine the effect of the present algorithm, we compare the potentials determined by the present scheme with original TVD scheme developed by Tanaka [1995]. In the present study, a substorm is simulated, which is produced by southward turning of northward IMF. The solar wind parameters used in the present study are as follows:

- IMF $(B_y, B_z) = (2.5 \, \text{nT}, \, 4.3 \, \text{nT}) \rightarrow (0 \, \text{nT}, \, -4.3 \, \text{nT})$
- Solar wind speed = 400 km/s
- Solar wind density = 5/cc
- Radius of inner boundary = 3.5 Re

Figure 1 shows the time variations of cross polar cap potentials for the substorm determined by both of simulation schemes. In this figure, IMF turned southward at $T=0$. The potentials increase with time and reach steady states in $\sim 40$ minutes. This means that the growth phase continued 40 minutes. Figure 2 shows the distributions of the ionospheric electric potential determined by both of the schemes and the difference between them. These distributions are determined in the middle of the growth phase of the substorm $(T=14 \, \text{min})$. Although the contour levels differ, the distributions determined by both of the schemes are almost the same. However, there is a obvious difference of the potential patterns in the nightside as shown in the right panel of Figure 2. Equation (8) shows that the difference of the potential between the original scheme and the present one is related to the ratio of the ionospheric conductivity and the Alfvén conductivity. In the case for Figure 2, this ratio is about 10:1, which corresponds to the ratio of the potential determined by the original scheme and the difference between the original scheme and the present one. As the small-scale distribution of the electric potential is effective for the present algorithm and the potential pattern in the nightside is complicated as compared to the dayside, the difference reaches its maximum at the nightside ionosphere.

In the present study, we have shown a test case for the substorm. Further calculations of various solar-wind condition are necessary to evaluate the present algorithm.

Figure 1 The variations cross polar cap potentials.

Figure 2 The ionospheric electric potentials determined by the MHD schemes at $T=14$ minutes. The left and center panels show the potential distribution determined by the original TVD scheme and that determined by the present scheme, respectively. The right panel shows the difference between them.